

# Magnetic Levitation Principles

## Introduction

Magnetic fields are used to describe forces at a distance from electric currents. These currents are of two types: (1) free, or Amperian, currents as drawn from a battery pack, power supply, or an electrical outlet and (2) bound currents as in permanent magnet materials. The forces come in three variations: a.) An electrical current feels a force from another current, b.) a current feels a force from a permanent magnet, and c.) a permanent magnet feels a force from another permanent magnet. This action at a distance is described by saying a magnetic field exists created by one of the bodies at the location of the other body. The magnetic field is the medium by which the force is transferred.

In this section, a brief discussion concerning the magnetic fields caused by magnetized materials (i.e., permanent magnets) is presented. By demonstrating that magnetic materials can be reduced to effective current distributions, this discussion forms the basis for calculating the forces on permanent magnets. The magnetic fields due to free current distributions are calculated next. These fields are used to calculate the forces felt by current-carrying conductors. Time-varying currents cause time-varying magnetic fields. These changing magnetic fields induce electric currents which, in turn, experience a force.

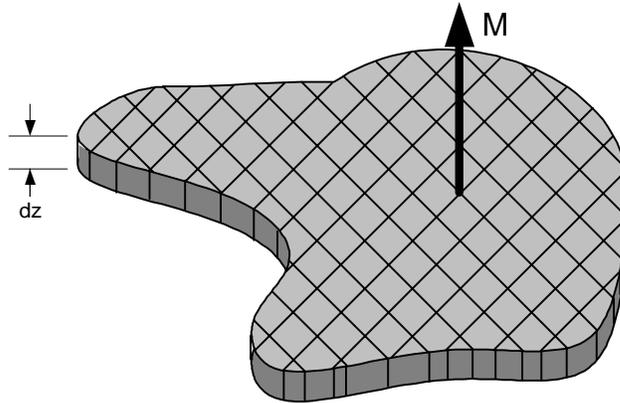
Maglev systems utilize the fundamental physics of electric currents experiencing forces-at-a-distance. These systems are most often described in terms of the interaction of electrical current with magnetic fields. Because the masses of the vehicles are large, large forces are required for magnetic suspension. These large forces are provided by the high magnetic fields of either large superconducting currents or small air gaps in normal ferromagnetic circuits.

A dictionary of common terms is provided at the end of the section and a reference list is included featuring books, Maglev studies and Maglev URLs.

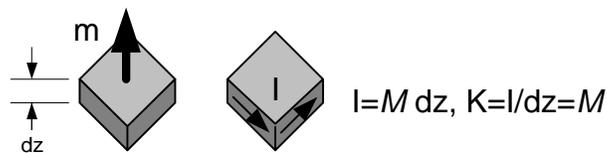
## Magnetic Fields Caused by Magnetized Materials

Electron spin is a quantum mechanical phenomenon. Its significance here is the fact that there is associated with the electron spin a magnetic moment of fixed magnitude. To determine the forces on magnetic materials, we use the fact that the magnetic moment of the electron spin *acts as if* it is a current loop.

A volume of magnetized material contains a very large number of aligned electron spins. This is illustrated schematically in the figure below. A grid pattern was superposed onto the material to indicate small volumes of materials which can be analyzed.

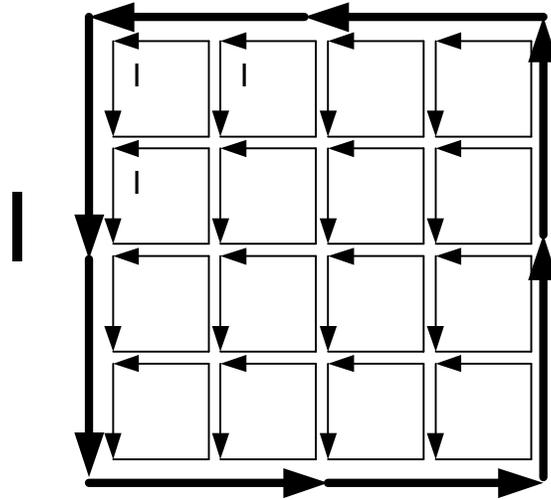


The figure below isolates two small volumes. The one on the left generates a magnetic field due to its magnetic moment,  $m$ . The one on the right generates a magnetic field by virtue of the current wrapping the volume (this is commonly referred to as a “current sheet.”), and otherwise ignores the presence of the magnetic material. ***The two magnetic fields are entirely equivalent*** (external to the material). The material property  $M$  describes the strength of the magnetic material and the amount of current required per unit distance of height. The parameter  $K$  describes the current per unit height within the current sheet. For modern high performance neodymium-iron-boron permanent magnets,  $K \sim 900,000$  amps/meter.

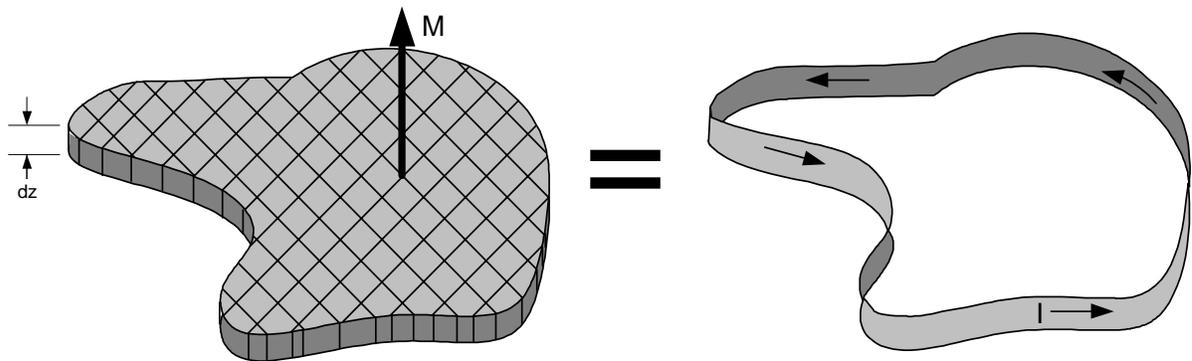


Equivalent as sources  
of magnetic field

Assembling many small volumes in juxtaposition, one can see in the figure below the result of substituting the current loop for the magnetic material; the internal currents cancel while the surface currents do not cancel. Hence, no matter the shape of the material, the external magnetic field can be exactly reproduced by a current stripe along the material perimeter. (This is true if the magnetization is uniform. If the magnetization is uniform then the currents are equal and cancel. If the magnetization is not uniform, then the equivalent current distribution can be calculated by  $J = \text{curl } M$ , where  $J$  is the volume current density.)



In summary, the magnetic field of a uniformly magnetized permanent magnet can be exactly reproduced by a current sheet along the perimeter. This equality is indicated graphically in the figure below. The magnitude of the current is proportional to the material thickness with the constant of proportionality dependent upon the material itself. This equivalence of magnetic fields is commonly exploited to calculate the forces on permanent magnet materials.



## Calculating Magnetic Fields

The Biot-Savart Law is the fundamental relationship between current and magnetic field:

$$d\vec{B} = \frac{\mu}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}, \quad (1)$$

where,

$d\vec{B}$  = differential magnetic field, tesla,

$\mu$  = permeability, Henry/m,

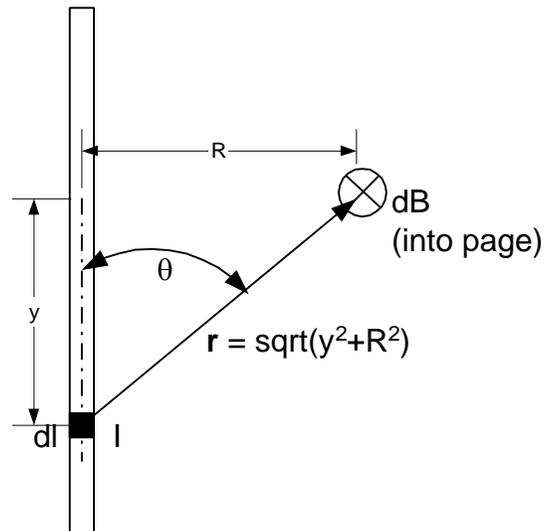
$I$  = current, amps,  
 $dl$  = differential length of current-carrying element, m,  
 $r$  = vector distance from current element to field point, m.

Two simple geometries for calculating the magnetic field are an infinitely long, straight conductor and a circular loop of conductor.

### ***Straight conductor***

For the straight conductor carrying current upwards along the y-direction, the magnetic field can be calculated at any point due to a differential current element as:

$$dB = \frac{\mu}{4\pi} \frac{I dl \sin \theta}{r^2} \quad (2)$$



By integrating along  $y$  from  $\pm$  infinity, the result is:

$$B = \frac{\mu I}{2\pi R},$$

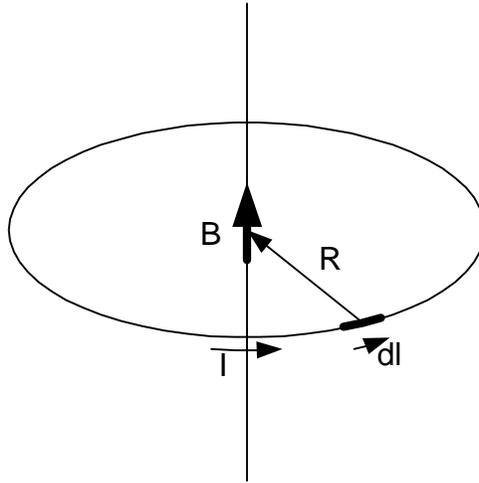
where the direction of the magnetic field is determined by the right hand rule (point the thumb of the right hand along the direction of current and the fingers curl in the direction of the magnetic field).

### ***Circular Loop: Center***

The magnetic field at the center of a circular loop of wire can also be calculated from the Biot-Savart Law. In this case, the angle between the current element and field point is a

constant ( $90^\circ$ ) so the vector cross product always yields unity. Then the magnetic field is:

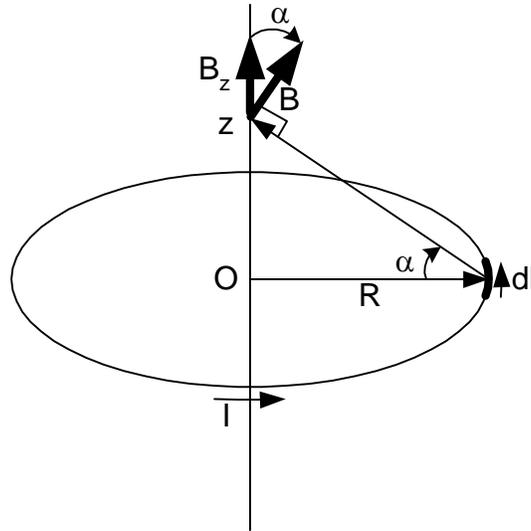
$$dB = \frac{\mu}{4\pi} \frac{I 2\pi R}{R^2} = \frac{\mu I}{2R}. \quad (3)$$



### ***Circular Loop: Anywhere Along Axis***

The magnetic field along the axis of a circular loop of wire can also be calculated from the Biot-Savart Law. In this case, the angle between the current element and field point is still a constant ( $90^\circ$ ) so the vector cross product always yields unity. The net magnetic field from any current element has vertical and horizontal components. However, as the current element follows the conductor path, the horizontal components of the field cancel while the vertical components add. Then the axial magnetic field is:

$$B_z = \frac{\mu}{4\pi} \frac{I 2\pi R}{z^2 + R^2} \cos \alpha = \frac{\mu I R}{2(z^2 + R^2)} \frac{R}{\sqrt{z^2 + R^2}} = \frac{\mu I R^2}{2(z^2 + R^2)^{3/2}}. \quad (4)$$



Due to the common occurrence of circular coils, this relationship is very important. Along the axis the magnetic field is purely vertical. For values of height,  $z$ , above the plane of the coil, which are small compared to the radius,  $R$ , ( $z \ll R$ ) the vertical magnetic field is insensitive to  $z$ . This is due to the finite radius of the coil. For heights  $z$  much larger than  $R$  ( $z \gg R$ ) the axial field decreases as the reciprocal third power of height. Off-axis, the radial magnetic field decreases as the reciprocal fourth power of height.

## Calculating Magnetic Forces

The force between current carrying conductors is given by the Lorentz Law:

$$d\vec{F} = I d\vec{l} \times \vec{B}, \quad (4)$$

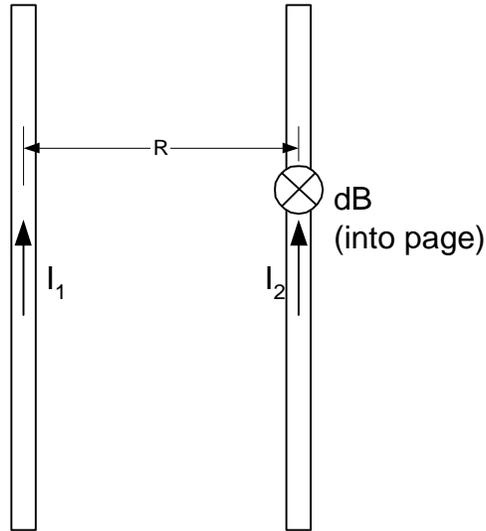
where,

$dF$  = differential force, Newtons.

One simple geometry for calculating the magnetic force per unit length is two infinitely long, straight conductors parallel to each other.

$$\frac{F}{l} = -\frac{\mu I_1 I_2}{2\pi R}.$$

The negative sign means the two parallel currents attract one another. If the direction of one of the currents were to be reversed (anti-parallel currents), the force would also reverse, and the currents would repel. Reversing *both* currents, of course, once again produces an attractive force.



To evaluate the magnetic field at off-axis points, the same formula (1) can be used. The mathematics quickly becomes intractable and the solution is usually implemented numerically. Alternatively, specialized computer aided design software can be used to calculate the magnetic field at arbitrary points in space. The two methods approach the calculation differently (the former is the idealized magnetic field numerically approximated while the other is the high fidelity numerical model of the detailed system). Either method will, of course, yield the same result.

The force due to a current-field interaction off-axis can be calculated according to (4). Thus, in the case of a Neodymium Iron Boron permanent magnet positioned above a coil, the magnet can be modeled as a current sheet of thickness equal to that of the magnet. This “current” creates a magnet field at the location of the coil conductors. The force on the coil and, by Newton’s third law, the force on the magnet is simply the product of the magnetic field, current and conductor path length.

Since the equivalent current of the permanent magnet is in the theta direction (circumferential around the magnet), the vector cross product in the force equation suggests that to get an axial force,  $F_z$  we must have a radial magnetic field,  $B_r$  ( $-\hat{z} = \hat{\theta} \times \hat{r}$ ). In developing the control system, it is convenient to express the radial magnetic field equation in the following form.:

$$B_r = \frac{C1}{(z + C2)^N},$$

where,

$C1$  and  $C2$  are constants depending upon the geometry of the permanent magnets and  $N$  is a parameter describing the decrease in magnetic field with increasing axial distance. For the case of the axial magnetic field, we saw above  $C1$  is the surface current density multiplied by the magnet thickness,  $C2$  is related to the square of the magnet radius, and  $N$  is three or less, depending upon the relative axial distance. The radial magnetic field

decreases more rapidly with distance than the axial magnetic field by approximately one more power of the denominator:  $3 < N < 4$ .

In order to suit the control law, the following form of the force equation for a magnet-coil interaction is sought:

$$F = \frac{K}{(z + D)^N} \cdot \frac{I}{I_{\max}}$$

The various magnet-coil interactions have been analyzed and the appropriate constants have been determined. Because of the inherent non-linearity of magnetic fields, these *constants* can vary when the excursion from the calculated system is large, i.e., the equations have been approximated by constant parameters in the region of interest. That is, at very large axial heights or very low heights the constants will differ from those calculated.

The form of the force is the same for interactions between permanent magnets due to the equivalent current concept discussed above. In this case, however, the current is not a free parameter for control but is determined by the geometry.

## Calculating Induced Currents

A time-varying magnetic field induces voltages in a closed loop according to Faraday's Law:

$$V = -N_t \frac{\partial \phi}{\partial t},$$

where,

V is the induced voltage around a closed loop,

$N_t$  is the number of turns of conductor around the loop, and,

$\partial \phi / \partial t$  is the flux rate of change through that loop.

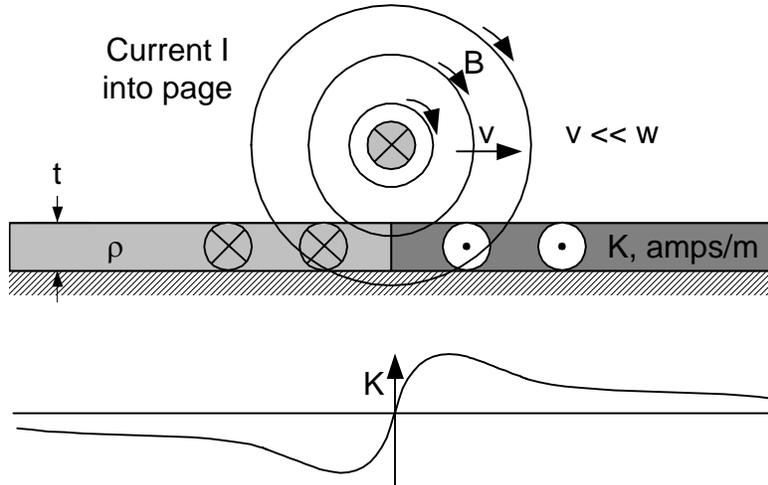
The negative sign in Faraday's Law is the manifestation of Lenz's Law. Lenz's Law states that when currents are induced in bodies due to a changing magnetic field, the currents are in such a direction as to cancel the change in magnetic field experienced by the body. For instance, if no field is present and suddenly a field is applied, the induced currents tend to circulate to cancel the magnetic field. If, however, the magnetic field has previously existed, removal of the magnetic field causes currents to flow in an attempt to maintain the field.

Consider a single current-carrying conductor moving relative to a conductive sheet. It can be shown [Magneto-Solid Mechanics, p. 339 ff] that there is a characteristic velocity

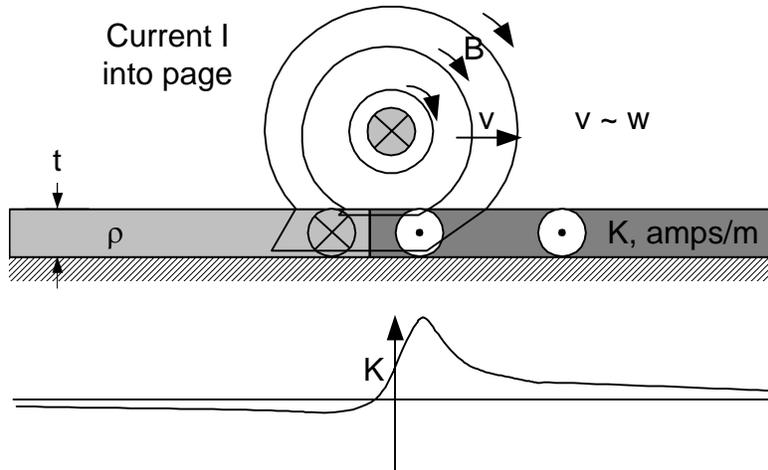
of the motion,  $w$ :  $w = \frac{2\rho}{\mu_0 t}$ , where,  $\rho$  is the sheet material resistivity,  $t$  is the thickness of

the sheet and  $\mu_0$  is the permeability of free space. At standstill, the magnetic field of the current will fully permeate the sheet conductor. The magnetic field lines will be perfect

circles about the current center. At very low speeds, ( $v \ll w$ ) the field still permeates the sheet and the field lines will still be very nearly circular. This situation is shown in the figure below, the induced current in the sheet is  $K$  amps/meter.

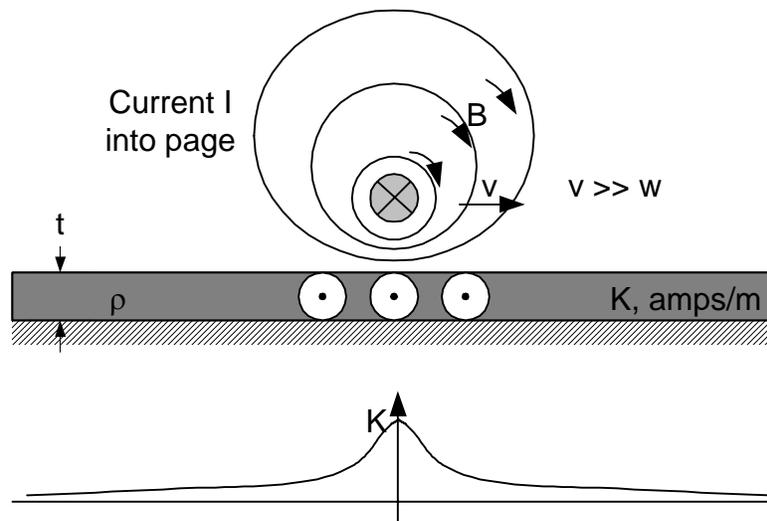


As the conductor speed is increased to approximately the characteristic velocity ( $v \sim w$ ), the movement of the magnetic field through the sheet causes induced currents to flow. According to Lenz's Law, these currents flow in such a manner so as to cancel the effect of the approaching field. Nevertheless, due to finite resistance, the magnetic field will still penetrate the conductive sheet to an extent and as the conductor leaves the region of magnetic field additional currents are induced to maintain the presence of the field. This situation is shown in the figure below. Notice the shear effect of the motion on the magnetic field.



When the speed is increased to substantially above the characteristics velocity, the conductivity of the sheet prevents the magnetic field from any significant penetration. The conductor is moving sufficiently fast that significant resistive dissipation does not

occur. Each section of sheet generates the exact required current to perfectly shield the interior of the conductive sheet from the magnetic field. This situation is shown in the figure below. Notice that the magnetic field lines do not enter the sheet.



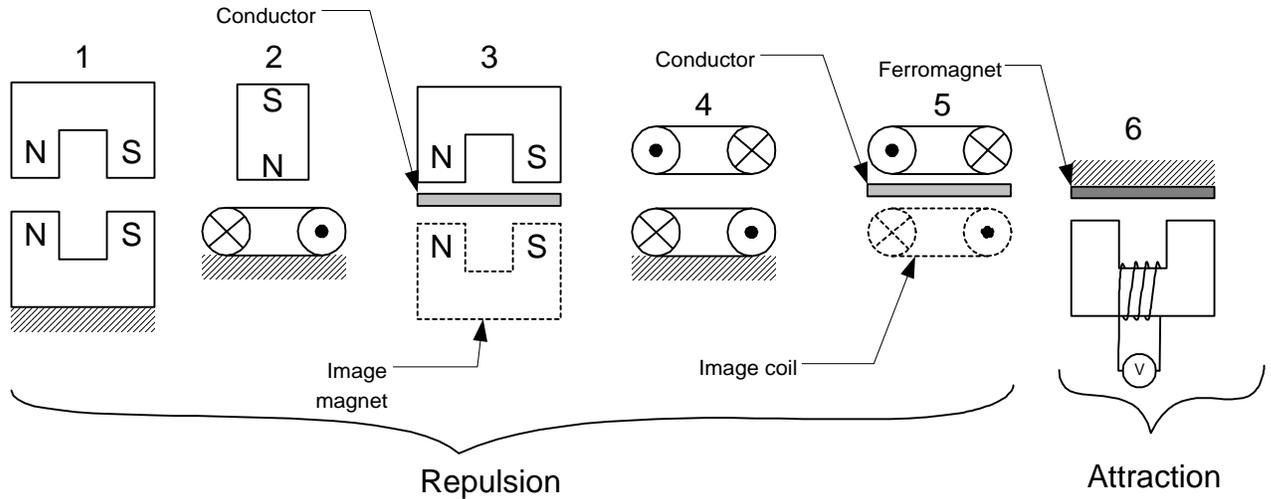
For application to the demonstration unit, the current element is the equivalent current of the permanent magnet edge. The permanent magnet flying above a rotating conductive sheet introduces several details different from that described above. The physical principle remains the same; that is, the currents are induced in the sheet in response to the apparent change in magnetic field, and these currents tend to cancel the change in magnetic field. The interaction of the (magnetic field from) the induced currents and the original permanent magnetic field produce a repulsion force which levitates the magnet. These fundamentals are the basis of several different configurations of Maglev systems.

## Maglev Applications

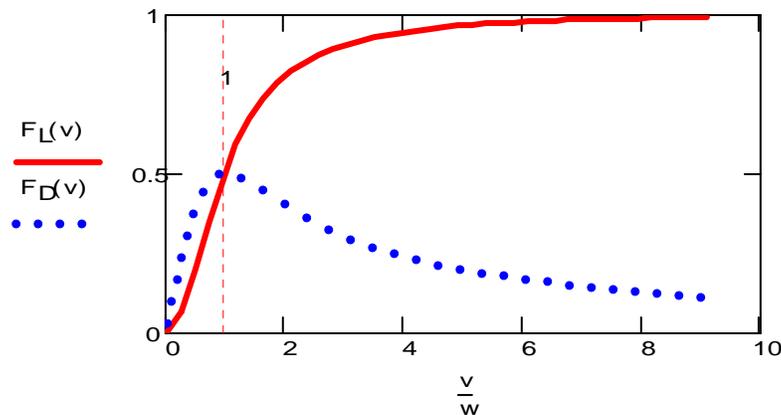
The figure below shows six Maglev arrangements. Five of the arrangements rely on repulsive forces. The lower elements are fixed (say, with respect to the earth) and the upper elements levitate. The first arrangement (permanent magnet like poles) is the common one for demonstrating like magnetic poles repel. The second and fourth arrangements (permanent magnet and/or superconducting magnet flying over a normal copper lower coil) is similar to that used for the Japanese superconducting Maglev system. The third and fifth systems are similar to the Magneplane system where permanent magnets or superconducting coils fly over normal sheet conductor. Notably, this has been variously proposed as an inexpensive method to attain levitation.

The sixth arrangement (electromagnet under a ferromagnet) is quite different and is the basis for the German Transrapid Maglev system. The fixed element is the upper ferromagnetic material and the lower electromagnet is actively controlled. If the current in the electromagnet is too large, the electromagnet feels a net upward force until it contacts the ferromagnetic material. If the current is too small, then insufficient force is

available and the electromagnet falls. Hence, the current in the electromagnet must be continuously adjusted to enable levitation without contact.



All Maglev designs share a common trait: while generating magnetic lift (in the direction perpendicular to travel) there is also generated magnetic drag (opposing the direction of travel). The details of the lift and drag forces, of course, depend upon the configuration, but the following figure (calculated for the conductor moving over the sheet, configuration 5 above) gives an idea of the variation in magnetic lift and drag with speed. The figure shows the drag peak and the reduction in drag as speed increases—in marked contrast to aerodynamic drag. The figure also shows the monotonic increase in lift force with increasing relative speed. Note that the lift force equals the drag force when the relative speed is equal to the characteristic velocity,  $w$ . Note also that this lift force is equal to 50% of the maximum lift force.



## Current Status of Maglev in the United States

Superconducting Maglev technology was initiated in the late 1960's and early 1970's in the United States in 1969 when Drs. James Powell and Gordon Danby of New York's Brookhaven National Laboratory invented the concept of a repulsive magnetic suspension using superconducting magnets. In the mid-1970's the US stopped Maglev development due to funding problems. Other countries, however, continued to develop Maglev and today have viable systems. In the early 1990's Maglev research was rekindled at a Federal government level. At various times, the Department of Transportation's Federal Railroad Administration and Federal Transit Administration, NASA, Department of the Air Force, and Department of the Navy have joined resources for the purpose of developing Maglev and Linear Motor technology. Although each agency had its own specific application in mind, a loose consortium seemed to provide the best bang for the buck. The situation has progressed where today there are several high speed (~300 mph) and low speed (~100 mph) regions in this country where Maglev is thought to be a viable alternative means of rapid public transportation—yet it is still unproven in the United States.

Germany's Transrapid vehicle has been extensively tested and has been proposed for use on several projects in this country. The Germans are presently constructing a Transrapid route from Hamburg to Berlin.

Japan is developing a system that uses superconducting magnets and is currently constructing a major test route that will ultimately be incorporated into a revenue-producing route. Approximately 80% of this system will be in tunnels cut into mountains. This has greatly increased the construction cost but has decreased the cost of land acquisition for the Maglev right-of-way.

### Dictionary:

Term	Definition
Eddy Currents	Induced currents in conductors by changing magnetic fields. Since the currents flow in closed paths within the materials, they are similar to eddies in rivers. In accordance with Lenz's Law, these currents flow in such a manner as to oppose the change in magnetic field inducing them. Frequently, eddy currents are undesirable, but they can be desired in cases of induction and microwave heating for industrial and consumer purposes.
EDS	Electrodynamic System
Electrodynamic System	Magnetic forces based upon repulsion from induced currents. Inherently stable can be lightly damped or even negatively damped at sufficiently high frequencies.
Electromagnetic System	Magnetic forces based upon attraction from applied currents. Inherently unstable and currents must be controlled.
EMS	Electromagnetic System
HSST	High Speed Surface Transportation (not as "high speed" as the name sounds). Japanese EMS Maglev.

Maglev	Generic term for magnetic suspension. Sometimes refers to both levitation(vertical forces) and guidance (side-to-side forces).
Magnetic Drag	Magnetic force opposing propulsion, due to resistive dissipation. Units are newtons.
Magnetic Field, <b>H</b>	Three-dimensional vector used to calculate the enclosed current as used in Ampere's Law. $\nabla \times H = \mu_0 J_{free}$ . This parameter is independent of the material properties. Units are Amperes/meter, A/m.
Magnetic Flux Density, <b>B</b>	Three-dimensional vector used to calculate the force on conductors and magnetic materials. Units are Wb/m <sup>2</sup> or Tesla, T. $\nabla \times B = \mu_0 (J_{free} + J_{bound})$ . Recall also $F = I dl \times B$ .
Magnetic Flux, $\phi$	$\phi = \iint \vec{B} \cdot d\vec{A}$ , the integral of the vector dot product of the flux density and area. Units are Webers, Wb.
Magnetic Lift	Magnetic force opposing gravity.
Permeability	$\mu=B/H$ , Ratio of flux density to magnetic field. Units are Henries/meter, H/m.
Reluctance	Ratio of total magnetomotive force to total flux through a circuit. Units are inverse Henries, H <sup>-1</sup> .
RTRI	Railway Technical Research Institute Japanese research arm to develop EDS Maglev
Superconductivity	The material property of the complete lack of electric resistance, obtained for special materials only under conditions of refrigeration. Two classes of materials are low critical temperature superconductors ( $T \leq 10K$ ) and high critical temperature superconductors (HTSC) ( $T \leq 77K$ ).
Transrapid	German EMS Maglev, Hamburg to Berlin route under construction.
Inductance, L	$L=N_t\phi/I$ , Total flux linked per unit current. Units are Henries, H.
Magnetomotive force, mmf	$MMF = \oint \vec{H} \cdot d\vec{l} = N_t I$ . Intermediate quantity used for magnetic circuits. Conceptually similar to the electromotive force or voltage of an electric circuit. Units of mmf are Ampere-turns.
Lenz's Law	A law stating the observed fact that induced currents resulting from a change in magnetic field are in such a direction as to oppose the change in magnetic field. Hence, if a magnetic field is increased, current is induced to cancel it. If the magnetic field is decreased, current is induced to preserve it.

## References:

### **Books:**

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2. Case Studies in Superconducting Magnets: Design and Operational Issues, Yukikazu Iwasa, Plenum Press, New York, 1994.
3. Magneto-Solid Mechanics, Francis Moon, Wiley & Sons, New York, 1984.
4. Electromagnetic Levitation and Suspension Techniques, B.V. Jayawant, Edward Arnold Publishers, London, 1981.
5. Electromagnetics, J.D. Kraus, McGraw Hill Companies, New York, 1992, p. 288.
6. Electricity & Magnetism, E. Purcell, McGraw-Hill Book Companies, New York, 1965.

### **Maglev Studies:**

7. "Study of Japanese Electrodynamic Suspension Maglev Systems", J.L. He, D.M. Rote, H.T. Coffey, Argonne National Lab, Argonne, IL, 60439, April 1994.
8. "New York State Technical and Economic Maglev Evaluation," Michael Proise, Grumman Space and Electronics Division, New York, June 1991.
9. "Electrodynamic Suspension and Linear Synchronous Motor Propulsion for High Speed Guided Ground Transportation," David Atherton, Canadian Maglev Group, CIGGT Report No. 77-13, September 1977.
10. "Conceptual Design and Analysis of The Tracked Magnetically Levitated Vehicle Technology Program Repulsion Scheme, Volume I – Technical Studies," Ford Motor Co., US DOT/FRA, Report PB247931, Feb. 1975.

### **Maglev Web Sites:**

11. <http://www.calpoly.edu/~cm/studpage/clottich/fund.html> Fundamentals of Maglev technologies and basic physics.
12. <http://www.rtri.or.jp/> RTRI Home page. Japanese EDS Maglev.
13. <http://www.transrapid-usa.com/> Transrapid Official Homepage. German EMS Maglev.
14. <http://www.meitetsu.co.jp/chsst/mecha.html> Mechanism of HSST. Japanese EMS.
15. [http://popularmechanics.com/science/research/1998/5/permanent\\_magnets/](http://popularmechanics.com/science/research/1998/5/permanent_magnets/) "Track to the Future" article. Descriptive article of one recent Maglev approach.
16. <http://www.llnl.gov/str/Post.html> "A New Approach to Levitating Trains - and Rockets." Elaboration of the previous link for a Maglev approach.